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Celtic Knots, Links and Mathematics

Abstraction is not the enemy, mathematics models the world around us, replacing complex things by simplified versions, a sphere for a cow for example. It helps sometimes and here is an example where it surely does !

Knots and links, used to illuminate medieval Bibles are impressive and intricate at first sight, but they can be modeled by a mere graph: some vertices linked by edges.

You are about to learn how this little shift in abstraction allows you to create, all by yourself, beautiful patterns and amazing Celtic knots.

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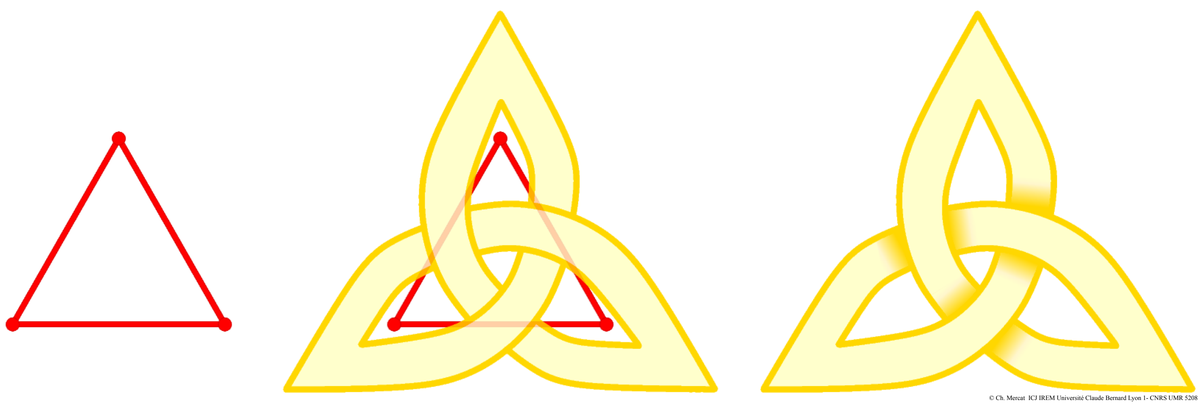
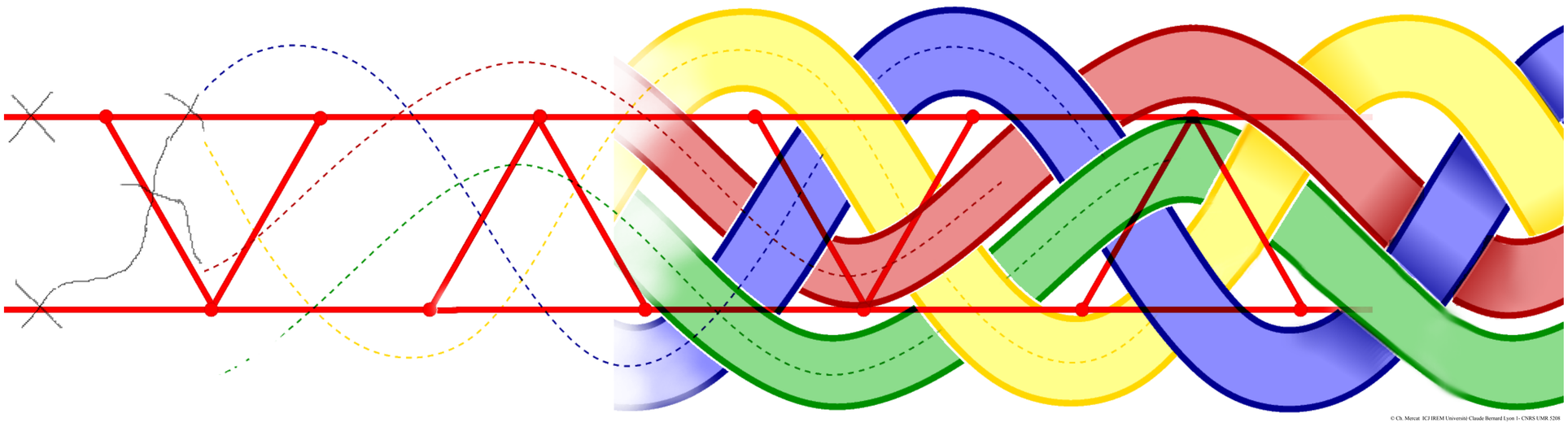


graph

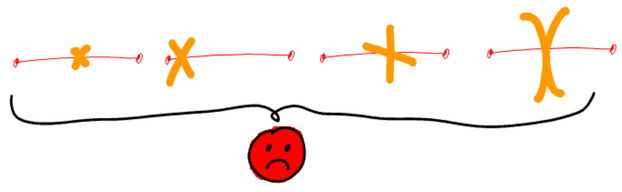
A graph is a set of points, called **vertices** linked by paths, called **edges**. It is **planar** when it can be drawn without edges intersecting.

Each graph encodes a link. For example the triangle encode the beautiful trefoil knot.

Once you have chosen a graph, the construction goes through three phases: place crossings at the middle of each edge, then link all bits of threads to one another, then decipher the over/under pattern, *et voilà!* Polish your artwork and be proud ! Let's walk though it slowly.

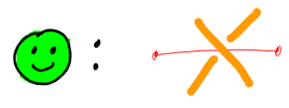


Drawing a link is like building a road, we begin setting the bridges. Right at the middle of each edge, draw a nice crossing, big enough, slanted somewhere between 30 and 60° with respect to the edge. Here we are, all the crossings are set.

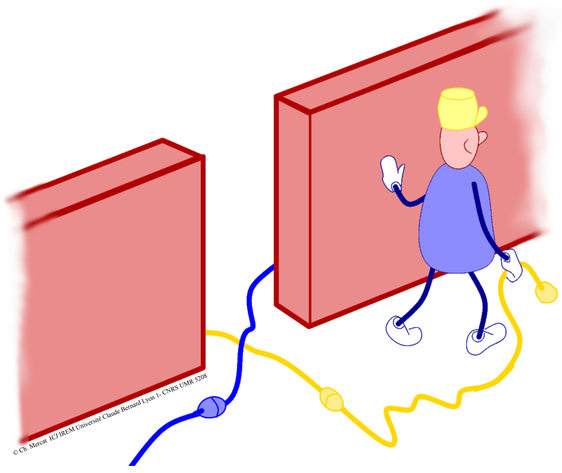


For each edge its midpoint crossing.

Each crossing on its edge.



Now let’s connect these bits of threads. The metaphora is to imagine you are in a dark maze, the edges of the graphs are its walls, that you cannot cross. In their middle, where the crossing occurs, a door allows you to go to the other side. Each bit of thread points in a given direction. Follow the wall until the next crossing points to you.



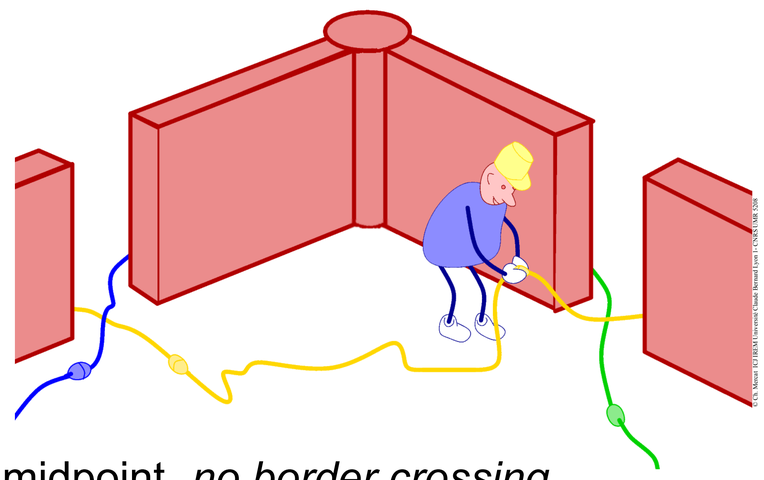
Follow the wall

With a bit of thread in your hand, follow the wall with your other hand, never letting it go. You will eventually reach another crossing.

Follow the wall, turn at the corner, follow the wall, here you are !

Once you reach a crossing, choose the bit pointing towards you. Don’t create a new one !

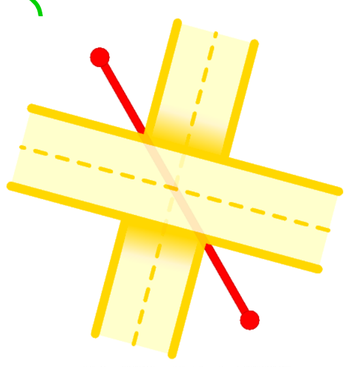
Once the path is clear, polish its trajectory a little.



Remember, edges are walls that you cannot cross, except at the midpoint, *no border crossing.* Moreover, each edge carries its crossing and each crossing takes place at the middle of an edge!

Now reproduce this little guide, of the same size and colour as an edge of your graph, with a reasonable bridge width (for an edge as long as your finger, the “roads” are as wide as well).

Then tear it off, align its edge with an edge of your graph, and it will tell you, among the two crossing paths, which one is above the other. Broaden your paths into full fledged roads.

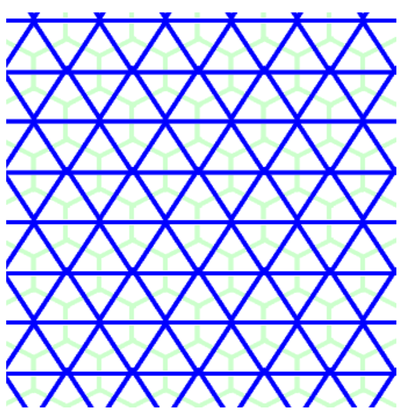


Tidy up and admire! Realize that without mathematics, it wouldn’t look as good!

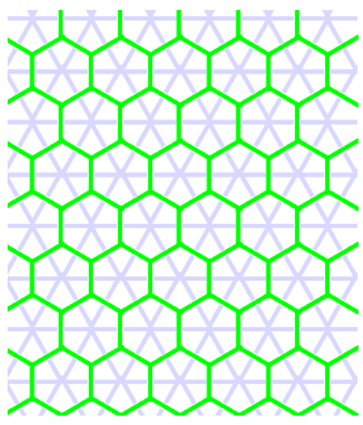
Designing a good looking knot is not that easy at first. You can lean on lattices, such as the square, the triangular or the hexagonal lattice.

Lattices

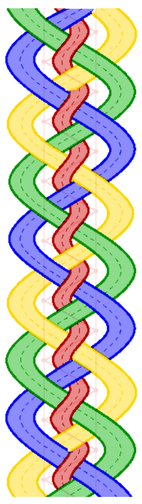
the **square** lattice



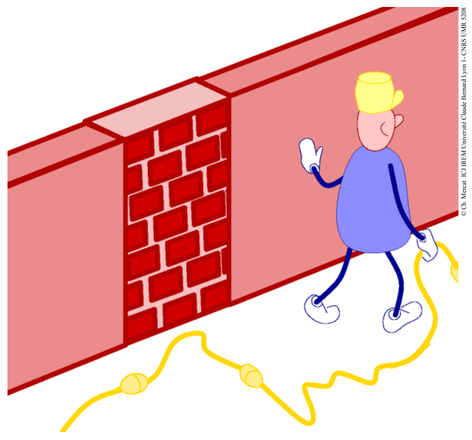
the **triangular** lattice



the **hexagonal** lattice



But lattices are too tame. You have to stir them a bit while preserving some regularity. Sprinkle your graph with well placed **walls**. From the point of view of the maze, they behave as if you’d closed the door, gluing together two vertices, or simply broke an edge and deleting it altogether. This can be encoded on the graph by thickening or slashing the edge.



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